Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- we have a normal distribution with =45 and 8.0.

Let X be the amount of time it take to complete the repair on a customer car.

To finish in one hour you must have X ≤ 50 so the question is to find Pr(X>50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - )/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 = 15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing centre are older than 44 than between 38 and 44. is F**ALSE**

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:-  Assume that *X*1 + *X*2 have same μ =1, σ = 2  
2 *X*1 = 2(1,22) = 2(1,4) = (2,8)  
*X*1 + *X*2 = (1 + 1, 22 + 22) = (2,8)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:-**  The Probability of getting value between a and b should be 0.99. So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).The Probability towards left from a = -0.005 (ie. 0.01/2). The Probability towards right from b = +0.005 (ie. 0.01/2).So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.Z=(X-μ) / σ For Probability 0.005 the Z Value is -2.57 (from Z Table).Z \* σ + μ = XZ(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:-** 95% of the probability lies between 196 standard deviations of the mean.

Thus range is =(12-1.96\*5,12+1.96\*5)

=($202M,$22.8M)

=(Rs.99M,Rs.1026M)

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:-**   P(Z<=(p-12)/5)=0.05

p-12/5= -1.644

p=12 – 8.22 = $3. 78 = rs.170.1

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:-** Second Division i.e. ~N(7,4²) has higher probability of making loss, due to higher variability.